Quark Distribution and Fragmentation Functions in the NJL model

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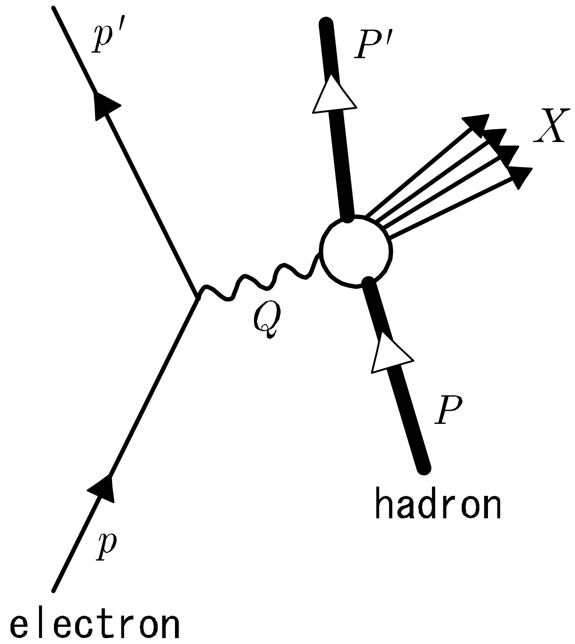
Structure functions

Structure function describes the internal structure of hadrons.

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\mathrm{d}E'} \propto F_2(Q^2,x) + 2F_1(Q^2,x)\tan^2\frac{\theta}{2}$$

At high Q^2

$$2xF_1(x) = F_2(x)$$



From parton model, structure function is expressed by "distribution function" of parton inside hadron.

$$F_2(x) = x\sum_q Q_q^2\left(f_q^h(x) + f_{\bar{q}}^h(x)\right)$$

f_q^h(x) is distribution function of quark q in hadron h

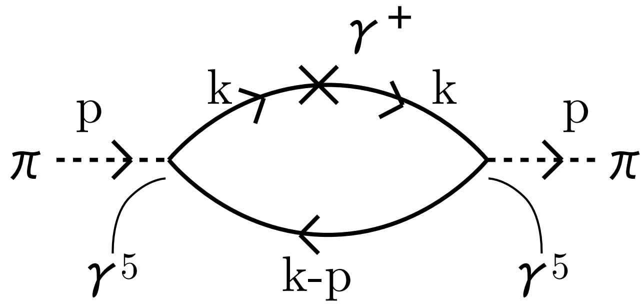
Distribution functions

Distribution function $f_q^{\,h}(x)$ shows momentum distribution of the quark "*q*" in the hadron "*h*".
It is defined as follows.

$$f_q^h(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \delta\left(k^+ - x\,p^+\right) \mathrm{Tr}[\gamma^+ \chi_q(k,p)]$$

with

$$\chi_q(k,p) = \int \mathrm{d}^4\xi\, \mathrm{e}^{-k\cdot\xi} \left\langle p \middle| \mathrm{T}\left(\overline{\Psi}_q(\xi)\Psi_q(0)\right) \middle| p \right\rangle$$



Number sum rule is obtained by integrate $f_q^{\,h}(x)$ over all x,

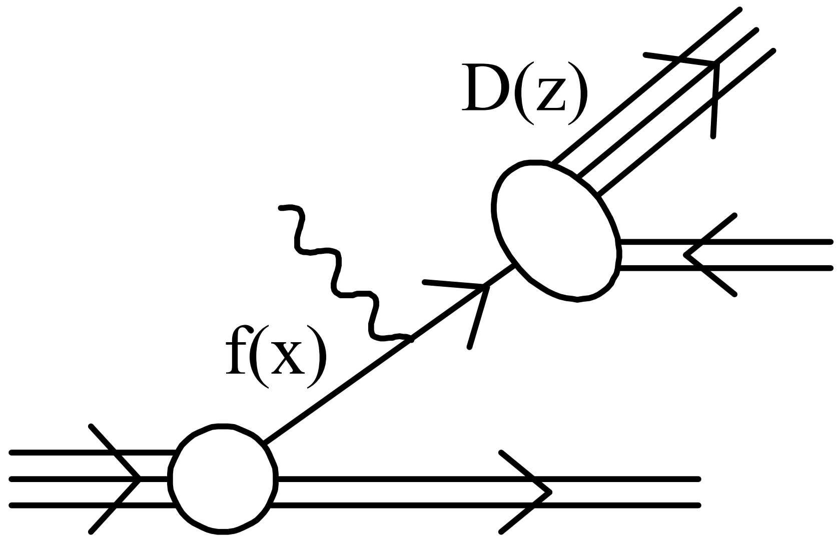
$$\int_0^1 f_q^h(x)\,\mathrm{d}x = N_{q/h}$$

Fragmentation functions

Hadron is generated from the quark that gets a large momentum from a virtual photon as shown in figure.

Fragmentation function $D(z)$ describes the process.

The purpose of this study is to calculate the distribution function $f(x)$ and the fragmentation function $D(z)$ within the same model.



Drell-Levy-Yan (DLY) relation between distribution and fragmentation function:

$$f_q^h(x) = \frac{1}{2}\hat{\sum_n}\delta(p\_x - p_- + p_{n-})\langle p|\bar{\psi}|p_n\rangle\gamma^+\langle p_n|\psi|p\rangle$$

$$D_q^h(z) = \frac{z}{6}\frac{1}{2}\hat{\sum_n}\delta(\frac{p_-}{z} - p_- - p_{n-})\langle p,\bar{p}_n|\bar{\psi}|0\rangle\gamma^+\langle 0|\psi|p,\bar{p}_n\rangle$$

From crossing and charge conjugation, one can derive the DLY relation:

$$\boxed{\pm\frac{z}{6}f_q^h(\frac{1}{z}) = D_q^h(z)}$$

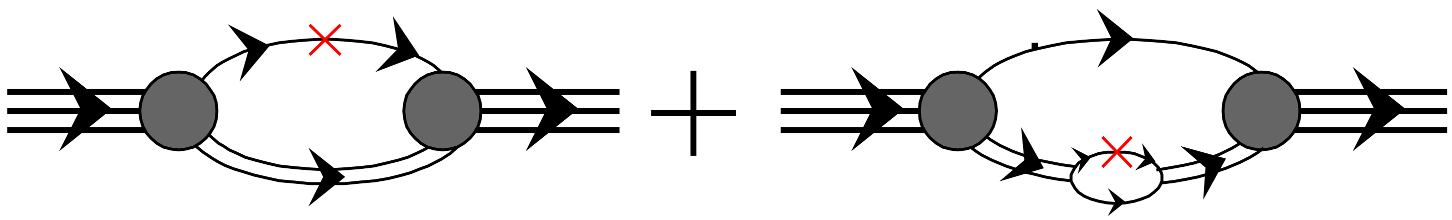
for bosons (+) and fermions (-). (Here 0 < z < 1)

origin of factor 1/6: Average over quark spin and color.

$f_q^h \equiv$ distribution of quark (*q*) in hadron (*h*); $D_q^h \equiv$ fragmentation of *q* into *h*

Numerical calculations: valence quarks

Use the Nambu-Jona-Lasinio (NJL) model as an effective quark theory, and a simple valence quark picture to calculate the quark distribution function from the Feynman diagrams (nucleon case)

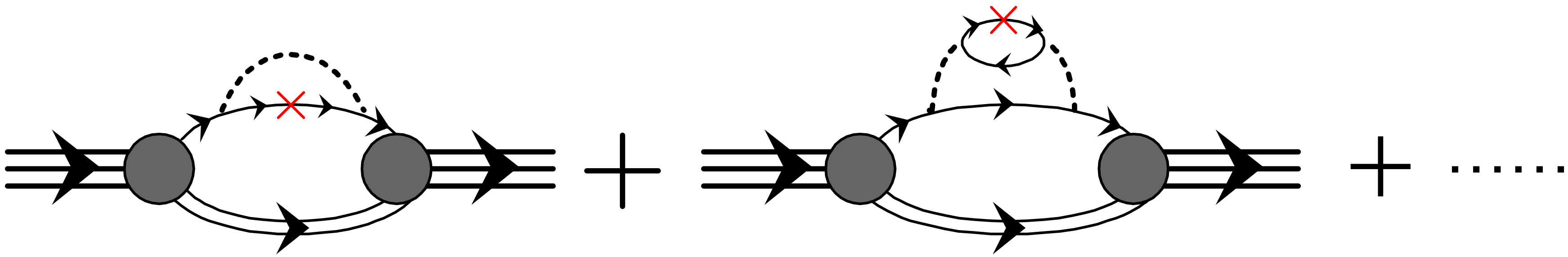


Nucleon ≡ quark + scalar diquark
Use pole approximation for diquark propagator.

Regularization: Transverse cut-off.
Parameters: Constituent quark mass $M = 0.4$ GeV, cut-off $\Lambda_{Tr} = 0.407$ GeV.

Numerical calculations: sea quarks

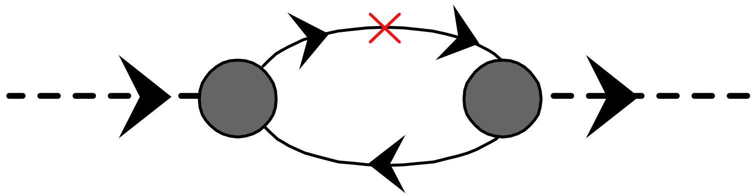
Include sea quark distributions as an effect of pion cloud around constituent quarks.
For the quark distribution function, we use the convolution formalism to evaluate diagrams like



Use pole approximation for diquark and pion propagators; on-shell (parent quark) approximation in the convolution integral.

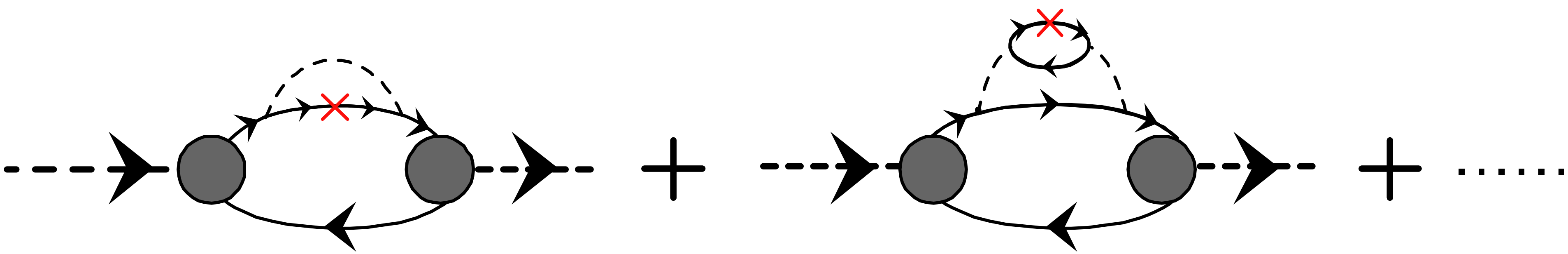
Numerical calculations: pion case

Feynman diagrams (valence picture)

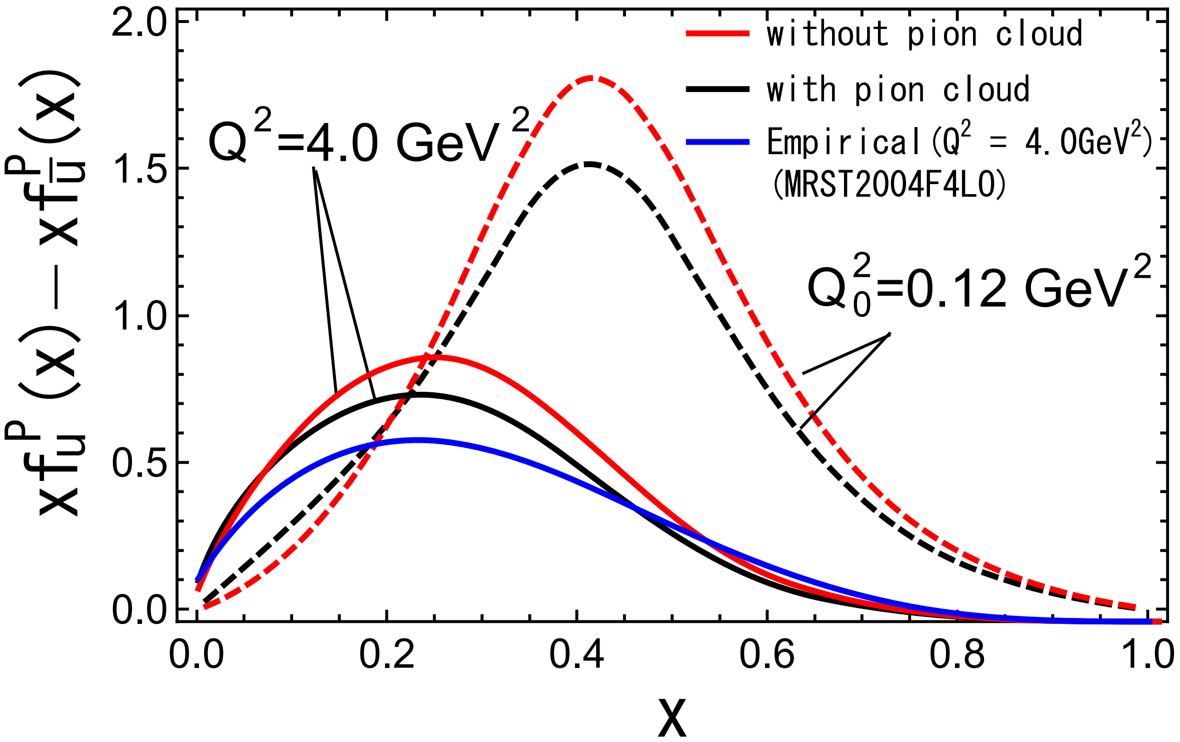


pion $\equiv$ quark + anti-quark

Including sea quarks:



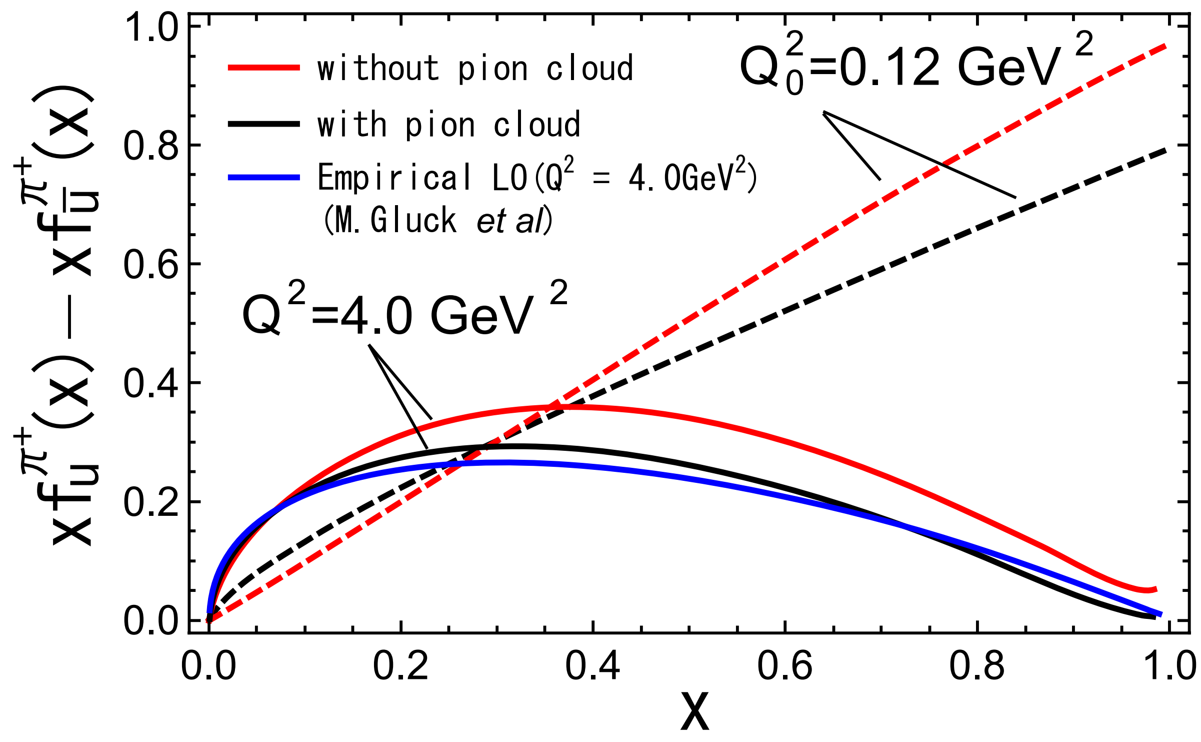
u valence quark distribution in proton



Dashed line = NJL result at $(Q_0^2 = 0.12\,\text{GeV}^2)$.
Solid line = evolved result at $(Q^2 = 4.0\,\text{GeV}^2)$.

Red line: without pion cloud
Black line:with pion cloud
Blue line:Empirical (MRST2004LO) $(Q^2 = 4.0\,\text{GeV}^2)$.

u valence quark distribution in π^+



Dashed line = NJL result at $(Q_0^2 = 0.12\,\mathrm{GeV}^2)$.
Solid line = evolved result at $(Q^2 = 4.0\,\mathrm{GeV}^2)$.

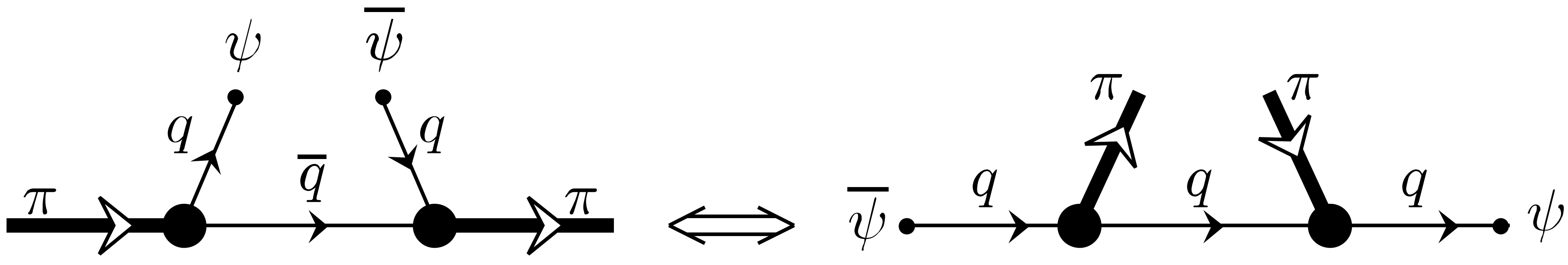
Red line: without pion cloud
Black line:with pion cloud
Blue line:Empirical LO (M.Gluck *et al*) $(Q^2 = 4.0\,\mathrm{GeV}^2)$.

We see:

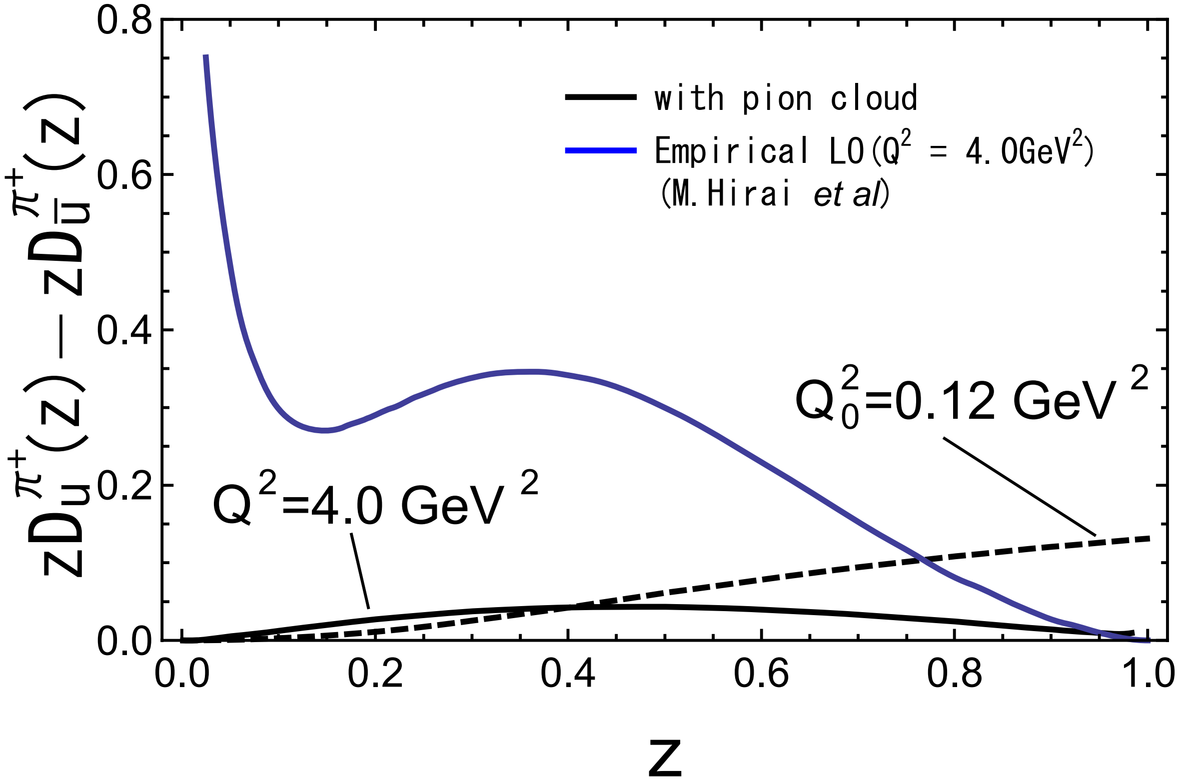
Simple models for pion and nucleon are successful for quark distribution functions.

Hope: Using the DLY relation we can also describe fragmentation functions.

$$\boxed{\pm\frac{z}{6} f_q^h\left(\frac{1}{z}\right) = D_q^h(z)}$$



u quark fragmentation into π^+



Dashed line = NJL result at $(Q_0^2 = 0.12\,\mathrm{GeV}^2)$.

Solid line = evolved result at $(Q^2 = 4.0\,\mathrm{GeV}^2)$.

Black line:with pion cloud

Blue line:Empirical LO (M.Hirai *et al*). $(Q^2 = 4.0\,\mathrm{GeV}^2)$.

Why are our results small?

From definition of fragmentation function:
$D_u^{\pi^+}(z)$ is the momentum distribution of π^+ in *u*.
In our field theory model, the number of pions in the quark N_u^π is given by 1 – Z_Q, where Z_Q is probability of "bare" quark without pion cloud.

Numerically, our pion number in the quark is small.

$$\int_0^1 dz D_u^\pi(z) = 0.23 = N_u^\pi$$

$$\int_0^1 z\, dz D_u^\pi(z) = 0.09 \quad \left(\Leftrightarrow \int_0^1 z\, dz D_{u,\mathrm{emp}}^\pi(z) = 0.45 \right)$$

→ Need more pions in the quark?

Conclusions

- DLY relation is based on crossing symmetry, and is very general. It expresses the fragmentation function by the distribution function.

- Simple effective quark models describe the distribution functions very well, but fail for the fragmentation functions.

- Possible reasons for failure:
  (i) We need more pions in the quark?
  (ii) Truncation of spectator states is problematic?
  (iii) What else?